

16. NONLINEAR EQUATIONS

Problem statement: Solution of equation: $f(x) = 0$

Symbolic solution: manipulation with expressions

Numeric solution: (i) separation of roots

(ii) iterative approximation of separate roots

COMMANDS

SYMS
SOLVE
SUBS
EZPLOT
PRETTY

ROOTS
POLY

POLAR

16.1 Symbolic Solution

Characteristics:

1. Symbolic solution is not always possible
2. Substitution allows conversion to numerical solution

% Example 16.1: Symbolic solution of equation: $ax^2+bx+c=0$

```
syms a b c x
r=solve('a*x^2+b*x+c=0'); pretty(simple(r))
r1=subs(r1,{a b c},{1 7 8})
```

% Example 16.2: Symbolic solution of equation: $\tan(x)+\sin(x)=2$

```
syms a b c x
r=solve('tan(x)+sin(x)=2'), double(r)
ezplot('tan(x)+sin(x)-2'); grid on
```

16.2 Numeric Solution

16.2.1 General Iterative Methods for Real Roots Estimation

Principle of the Newton Method:

1. Estimation of the initial approximation of the root: $x(1)$
 2. Definition of the tangent to function $f(x)$ at point $[x(1), f(x(1))]$
- $$y - f(x(1)) = f'(x) (x - x(1))$$

3. Intersection of the tangent with x-axis: $x(2) = x(1) - f(x(1))/f'(x(1))$

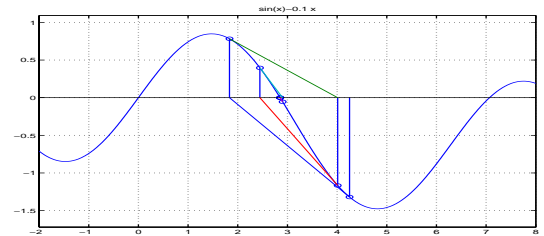
%%% Example 16.3: Solution of equation $\sin(x)-0.1x=0$ by Newton method for

%%% initial approximation $x(1)$, accuracy eps and maximum number of iterations M

```
x(1)=4.25; eps=0.0001; M=10;
for i=2:M
    x(i)=x(i-1)-f(x(i-1))/fd(x(i-1));
    if (abs(x(i)-x(i-1))<eps), break; end
end
ezplot('sin(x)-0.1*x', [-2 8]); grid on
hold on; stem(x,f(x)); hold off
line([x(1:end-1);x(2:end)], [f(x(1:end-1));...
    zeros(1,length(x(2:end)))]);
```

%%%

```
function y=f(x)    function yd=fd(x)
y=sin(x)-0.1*x;   yd=cos(x)-0.1;
```

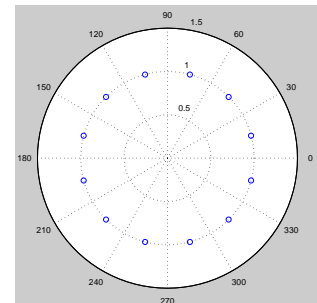


16.2.2 Roots of Algebraic Equations

Equation definition: $f(x) \equiv c_1x^n + c_2x^{n-1} + \dots + c_{n+1} = 0$

%%% Example 16.4: Solution of algebraic equation $x^{12}+1=0$

```
c=[1 zeros(1,11) 1];
r=roots(c)
polar(angle(r), abs(r),'o'); grid on
cc=poly(r)
```



EXAMPLES 16

16.1 Evaluate symbolic and numeric solution of a selected nonlinear equation

16.2 Find and plot roots of selected algebraic equations